Sensitivity of the Kurtosis Statistic as a Detector of Pulsed Sinusoidal Radio Frequency Interference in a Microwave Radiometer Receiver

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Abstract—**Radio frequency interference (RFI) from anthropogenic sources in microwave radiometers detecting geophysical parameters is both common and insidious. As this RFI is always additive to the brightness, the presence of undetected RFI can bias the geophysical parameter retrieval. As radiometers have the most sensitive receivers operating in their band, low levels of RFI are both significant and difficult to identify. The kurtosis statistic is a tool being explored as a means of detecting low level RFI in microwave receivers. The performance of the kurtosis statistic as a detector of RFI is introduced.**

Keywords-Detectors, digital radio, interference suppression, microwave radiometry.

I. INTRODUCTION

The kurtosis statistic of a sample population is the 4^{th} central moment normalized by the square of the 2^{nd} central moment (variance). When applied to the pre-detected output voltage of a microwave radiometer, the kurtosis can be used to detect the presence of low levels of RFI [1]. This is because the RFI-free signal generated at the output of the radiometer is composed only of thermal brightness of the scene and the noise generated by the radiometer receiver, while RFI is often generated by non-thermal mechanisms. The thermal noise voltage is Gaussian distributed, which has a fixed kurtosis value of 3 that is independent of the brightness temperature. Since the kurtosis statistic is calculated from a finite sample, the kurtosis of thermal noise has a mean of 3 and a variance of $24/N$, where N is a sufficiently large number of independent samples used to calculate the kurtosis. As an example of non-thermal RFI, we will consider a pulsed sinusoid, which is representative of a typical air traffic control or early warning radar transmission. This waveform, combined with the thermal noise, is very distinct from thermal noise, and has a distinct kurtosis value which depends on the strength of the RFI relative to the thermal noise and on the pulsed sinusoid duty cycle. The kurtosis decreases toward 1.5 as the 100% duty cycle (CW) RFI strength grows. Conversely, the kurtosis grows larger than 3 as the RFI duty cycle gets very short. The total brightness contribution of a short sinusoidal pulse can be quite small, yet be detectable with the kurtosis statistic. Despite the obvious differences in the voltage distribution when the RFI duty cycle is 50%, the kurtosis based algorithm is blind in this situation, regardless of RFI strength, because the kurtosis of a 50% duty cycle pulsed sinusoid is identical to the RFI-free kurtosis value of 3. However, operational radars typically transmit with very short duty cycles, and so we expect to be able to use the kurtosis to detect weak, short pulses that may corrupt a radiometric measurement. Moreover, since the sample kurtosis and the sample variance drawn from a Gaussian population are uncorrelated, the kurtosis provides an unbiased means of detecting RFI, unlike other methods that rely only upon the variance (power) of the pre-detected radiometer output voltage.

With a calculation of the false alarm rate and the probability of detection of a pulsed sinusoid, it can be shown that the minimum detectable pulsed sinusoidal power depends on the inverse fourth root of N. Thus, the minimum detectable RFI power level can be reduced by increasing the observation bandwidth and/or integration period. The minimum detectable RFI power decreases with increasing N slower than the noise equivalent radiometric uncertainty (which depends on the inverse square root of N). However, the sensitivity of the kurtosis approach to pulsed sinusoidal RFI can be dramatically increased by dividing the radiometer bandwidth into subbands. The effect of M equal sized subbands on the minimum detectable pulsed sinusoidal RFI in a radiometer system is the same as multiplying the number of independent samples by the cube of M. In addition, when RFI is detected, subbanding permits an unbiased calculation of the scene brightness from those subbands that are not corrupted by RFI, albeit with increased uncertainty. Both the calculation of moments and the division of the bandwidth can be readily achieved within a field-programmable gate array (FPGA) in a digital receiver. The net effect is that, under appropriate conditions, a digital kurtosis algorithm should detect short pulses of sinusoidal RFI at brightness contribution levels of a fraction of the radiometric uncertainty.

This paper presents the results of some validation activities for the claims made above. A detailed description of the derivation of theoretical behavior of the kurtosis statistic can be found in [2].

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II. Theoretical Background

A. The Kurtosis Statistic

The kurtosis R of the predetected voltage v is defined as the 4th central moment divided by the 2nd central moment squared, ie.

$$R = \frac{\left\langle \left(v - \langle v \rangle\right)^4 \right\rangle}{\left\langle \left(v - \langle v \rangle\right)^2 \right\rangle^2} \qquad (1)$$

Implicit in (1) is the knowledge of the probability density function of v. Rice [3] derived the instantaneous probability distribution of the voltage of a sine wave of amplitude A contaminated with Gaussian noise of variance σ^2. His expression can be easily extended to the case of a pulsed sinusoid with duty cycle d, where $0<d<1$:

$$p(v) = \frac{1}{\sqrt{2\pi}\sigma} e^{-v^2/2\sigma^2} \left(1 + d\sum_{k=1}^{\infty} \frac{1}{(k!)^2} \left(\frac{A}{2\sigma}\right)^{2k} He_{2k}\left(\frac{v}{\sigma}\right)\right) \qquad (2)$$

where

$$He_{2k}(x) = \sum_{m=0}^{k} \frac{(-1)^m}{2^m} \frac{(2k)!}{m!(2k-2m)!} x^{2k-2m} \qquad (3)$$

is a Hermite polynomial of even order [4]. When d=0 or A=0, (2) reduces to the Gaussian probability density function. Rice's expression follows when d=1.

Since the power (variance) of the pulsed sinusoid is $dA^2/2$ and the power in the noise is σ^2, we can define a pulsed sinusoid to noise ratio, S, and furthermore we can also define an equivalent radio brightness of the pulsed sinusoid, T_{ps}:

$$S = \frac{dA^2}{2\sigma^2} = \frac{T_{ps}}{T_{sys}} \qquad (4)$$

With these mathematical tools at our disposal, it is straightforward to show that the mean value of the kurtosis is given by

$$\overline{R}(S,d) = 3\frac{\left(1 + 2S + \frac{1}{2d}S^2\right)}{(1+S)^2} \qquad (5)$$

In the absence of interference, S=0, and the kurtosis has a value of 3. In the CW limit (d=1), $\overline{R}$ ranges from 3 to 1.5 as the power in the sinusoid increases. However, as $d \rightarrow 0$ but $S>0$, which corresponds to very short radar pulses, $\overline{R} > 3$. These deviations from 3 enable the detection of the presence of pulsed sinusoid.

Operationally, we perform the moment calculations in (1) with a finite number of samples, $N=B\tau$, where B is the radiometer bandwidth and τ is the radiometer integration time. An implicit assumption is made that the radiometer sampling rate is sufficiently slow that all of the samples are independent. Because we are using a finite number of samples in the direct calculation of the kurtosis, the kurtosis under a given sinusoid to noise ratio, S, and duty cycle, d, has both a mean value, $\overline{R}$, and a variance, σ_R^2, around that value. For a sufficiently large value of N, which is roughly 50kSa or more, the variance of the kurtosis in the absence of RFI (denoted σ_{R0}^2) is $24/N$. Because we are most interested in RFI which is of sufficiently low power that it is difficult to otherwise detect, we will assume for the purpose of this paper that the variance of the kurtosis is always very close to $24/N$. A more thorough analysis is available in [2].

B. Detecting RFI using the Kurtosis Statistic

Once the kurtosis is calculated for a given radiometric observation, we compare it with the threshold values of the kurtosis to decide if it is sufficiently close to 3 to be considered free of RFI. With the mean and standard deviations of the kurtosis known both in the presence and absence of pulsed sinusoidal RFI, we can define a false alarm rate (*FAR*) and probability of detection (*PD*) for these threshold values. Let us denote the threshold values as R_{tha} and R_{thb}, employing the subscript "a" and "b" for kurtosis threshold above and below 3, respectively. Then, $FAR=FAR_a+FAR_b$, where

$$FAR_{a,b} = \tfrac{1}{2}\left(1 \mp erf\left(\frac{R_{tha,thb} - 3}{\sqrt{2}\sigma_{R0}}\right)\right) \qquad (6)$$

where $erf(x)$ is the error function of x. Also,

$$PD_{a,b} = \tfrac{1}{2}\left(1 \mp erf\left(\frac{R_{tha,thb} - \overline{R}}{\sqrt{2}\sigma_{R}}\right)\right) \qquad (7)$$

where PD_a applies to short pulse sinusoidal (radar-like) RFI and PD_b applies to CW RFI.

To explore the minimum detectable RFI, let us fix the *FAR*, and identify S_{min} as the minimum S for which $PD=1-FAR$. This is the same as locating the kurtosis threshold the same number of standard deviations, z, away from the means of the RFI-free and RFI-contaminated kurtosis pdfs:

$$R_{tha,thb} = 3 - z\sigma_{R0} = \overline{R}(S_{min}, d) + z\sigma_R \qquad (8)$$

Solving (8) for the constant parameter z using (5), we obtain

$$z = \frac{3S_{min}^2 \sqrt{\frac{N}{24}}\left(1 - \frac{1}{2d}\right)}{2(1+S_{min})^2} \qquad (9)$$

Equation (9) demonstrates that as N grows very large, S_{min} is proportional to $N^{-1/4}$. That is, the minimum detectable RFI, like the noise equivalent radiometric uncertainty (NEΔT), can be reduced by increasing the number of samples. However, as the NEΔT is proportional to $N^{-1/2}$, the minimum detectable RFI decreases somewhat slower than the NEΔT as the bandwidth or integration time increases.

Division of the RF bandwidth into subbands is an approach to salvage some observations when RFI is narrowband. Subbanding has been used to detect RFI by comparing the power in each subband [5,6]. This division into M subbands also enhances the sensitivity of the kurtosis statistic to RFI. Because the RFI is presumed to be narrowband, the RFI signal strength falling in a subband is the same as it would be in the full band. However, the noise power is proportional to bandwidth, and so the sinusoid to noise ratio for the subband, S_{sb}, is enhanced by a factor of M over that for the full band: $S_{sb}=MS$. Meanwhile, the number of independent samples for each subband, N_{sb}, is reduced by a factor of M: $N_{sb}=N/M$. As the sensitivity analysis for a subband is the same as for the full band, we can conclude that the minimum S_{sb} is proportional to $N_{sb}^{-1/4}$. Thus, the S_{min} for entire system is proportional to $(M^3N)^{-1/4}$, up to some maximum value of M at which the spectrum of the RFI pulses are no longer contained within one subband.

III. Experimental Setup

To test the capabilities of the kurtosis statistic for RFI detection, a benchtop digital radiometer has been constructed into which a controlled sinusoidal pulse is injected. A pulsed sinusoid was created in an Agilent N6030A Arbitrary Waveform Generator (AWG), which is part of an L-band version of the Correlated Noise Calibration System (CNCS) [7]. One CNCS L-band RF head was used to provide a 303.9K reference load and a 73.3K Coldfet. The pulsed sinusoid was injected on top of the Coldfet brightness. The AWG was programmed to create a pulsed sinusoid at 320.75MHz with 0.1%, 1%, 10%, 50% and 100% duty cycles. The pulsed sinusoid amplitude was controlled with attenuators, in 6dB steps, inserted at the AWG output, prior to upconversion within the RF head to 1412.75MHz. The CNCS LO is at 1092MHz.

The analog front end to this benchtop radiometer was provided by the RFI Detection and Mitigation (DetMit) Testbed [8], in which the RF was filtered to a 20MHz bandwidth and downconverted by an LO at 1440.5MHz to an IF with a center frequency of 27MHz. This portion of the radiometer was not thermally controlled. The digital back-end was provided by the Agile Digital Detector (ADD) [9], which digitized the incoming IF at 8 bits and a rate of 110MSa/s. The data was processed in an onboard FPGA, where the number of occurrences of a given digitized level was counted in each 36ms integration period. The ADD FPGA also performed this same histogram generating operation on 8 subbands, each 3MHz wide. The pulsed sinusoid falls within subband 5, and 0.75MHz outside of subband 4.

IV. Results

Figure 1 shows a comparison of the histograms collected by ADD in subband 5 compared with those predicted by eqn. (2). The background Gaussian noise level was kept constant and the pulsed sinusoidal RFI, when present, was also held at a constant amplitude. The duty cycle shown includes 100%, 50% and 10%; below 10% it is difficult to visually distinguish the curves with RFI from the Gaussian pdf. The agreement between the data and eqn. (2) is excellent.

Figure 2 shows the kurtosis calculated from the histogram data, compared with the expected kurtosis from eqn. (5). The agreement between experimental data and theoretical predictions are again excellent. The CW (d=100%) kurtosis values are less than 3, and the short duty cycle kurtosis values are greater than 3. Note that the 50% duty cycle pulsed sinusoid has a kurtosis of 3, regardless of the RFI strength. Such RFI will not be flagged by a kurtosis based algorithm, but few such sources are expected to occur in practice.

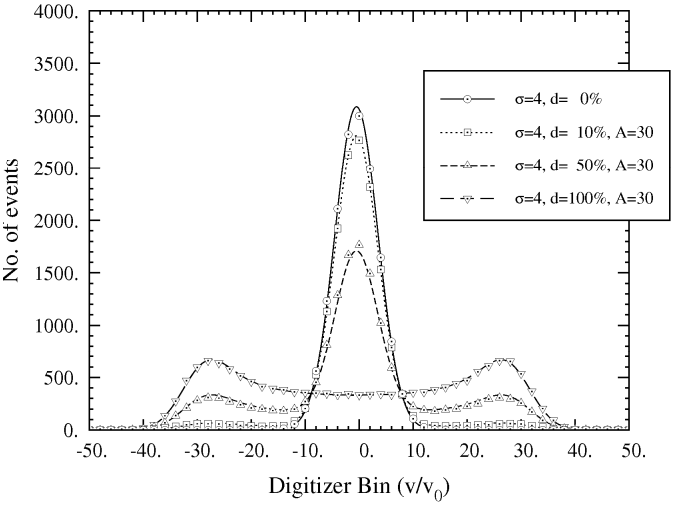

Figure 1: Instantaneous voltage pdf for constant Gaussian noise level and constant pulsed sinusoid amplitude but varying duty cycle. Curves are from eqn. (2).

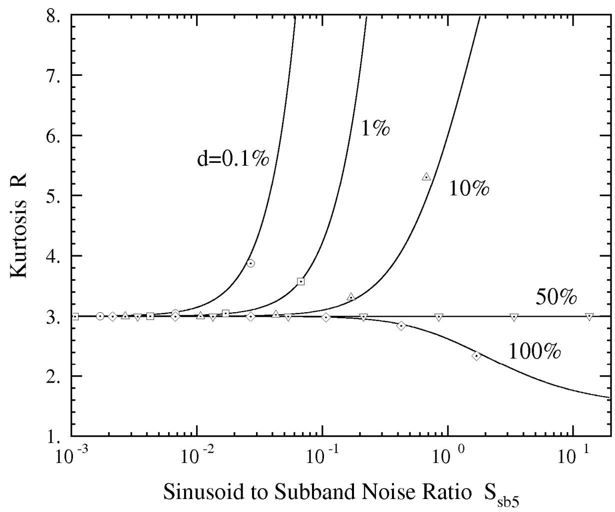

Figure 2: Kurtosis in subband 5 as a function of sinusoid to noise ratio and duty cycle. Curves are from eqn. (5).

Figure 3 shows an expansion of the data in Fig. 2 around the region R=3. Figure 3 also shows the standard deviations of both the sinusoid to noise ratio and the kurtosis. This figure demonstrates the power of a kurtosis based method of detecting short pulsed sinusoidal RFI. The 0.1% duty cycle pulse, approximately the duty cycle of ARSR-1 radars, is unambiguously detectable at a sinusoid to noise ratio of less than 0.01.

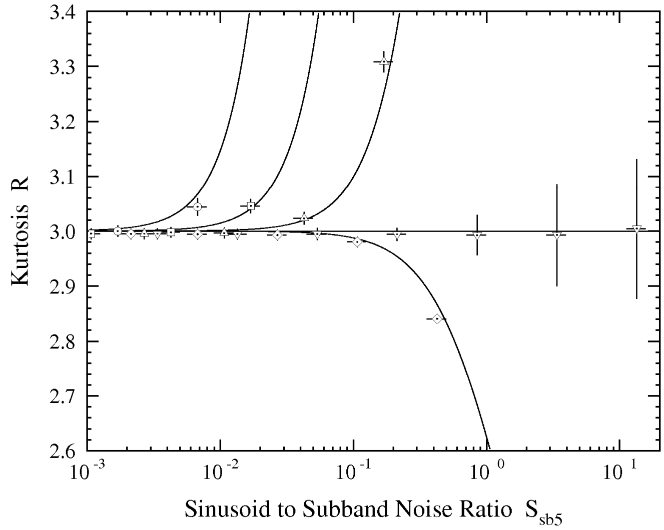

Figure 3: Close-up of the data in the previous figure, with 1 standard deviation error bars.

CONCLUSIONS

The kurtosis of the pre-detected voltage is presented as a means of detecting the presence of pulsed sinusoidal RFI in radiometric observations. The pre-detected voltage due to thermal emission obeys the Gaussian pdf, for which the kurtosis is 3, regardless of the variance (brightness) of the signal. When pulsed sinusoidal RFI is added to the thermal noise, the kurtosis statistic can deviate significantly from 3, indicating the presence of RFI. CW RFI tends to drive the kurtosis to less than 3, while short duty cycle pulses, such as those from radars, drive the kurtosis to exceed 3. A pulsed sinusoid with 50% duty cycle has an expected kurtosis of 3 regardless of sinusoid strength and thus cannot be detected with the kurtosis.

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